1 Elementarmathematik - Lösungen

1.1 Binomische Formeln

1. Wenden Sie binomische Formeln zum Umformen folgender Terme an!
$\boxed{a^2 \pm 2ab + b^2 = (a \pm b)^2}$ $\boxed{a^2 - b^2 = (a+b)(a-b)}$

   (a) $4x^2 + 12xy + 9y^2 = (2x+3y)^2 \quad (a = 2x,\ b = 3y)$

   (b) $a^2 - 4ab + 4b^2 = (a-2b)^2$

   (c) $(\sqrt{a}+2b)(\sqrt{a}-2b) = a - 4b^2$

   (d) $\dfrac{a^2-4b}{a+2\sqrt{b}} = \dfrac{(a+2\sqrt{b})(a-2\sqrt{b})}{a+2\sqrt{b}} = a - 2\sqrt{b}$

2. Führen Sie für folgende Terme eine quadratische Ergänzung durch!

   (a) $x^2 + 6x + 5 = x^2 + 6x + 3^2 - 3^2 + 5 = (x+3)^2 - 4$

   (b) $a^2 - 4a + 3 = a^2 - 4a + 2^2 - 2^2 + 3 = (a-2)^2 - 1$

   (c) $3y^2 - 3y + 1 = 3(y^2 - y) + 1 = 3\left(y^2 - y + (\frac{1}{2})^2 - (\frac{1}{2})^2\right) + 1 = 3(y - \frac{1}{2})^2 + \frac{1}{4}$

1.2 Lösen von Gleichungen und Termumformung

1. Lösen Sie folgende Gleichungen!

   (a) $\dfrac{x}{3} - \dfrac{9}{x} - 2 = 0$
   $\frac{1}{3}x^2 - 9 - 2x = 0 \quad \rightarrow \quad x^2 - 6x - 27 = 0 \quad \rightarrow \quad x_1 = -3,\ x_2 = 9$

   (b) $\dfrac{x+1}{x-1} = \dfrac{x-3}{x+1}$

   $$\begin{aligned} (x+1)^2 &= (x-3)(x-1) \\ x^2 + 2x + 1 &= x^2 - 4x + 3 \\ 6x &= 2 \\ x &= \tfrac{1}{3} \end{aligned}$$

   (c) $\dfrac{3x+1}{x-1} = \dfrac{x-1}{x+1}$

   $$\begin{aligned} (3x+1)(x+1) &= (x-1)^2 \\ 3x^2 + 4x + 1 &= x^2 - 2x + 1 \\ 2x^2 + 6x &= 0 \\ x_1 = 0 \quad &, \ x_2 = -3 \end{aligned}$$

   (d) $2x + 6 + \dfrac{1}{x+3} = 3x - 5$

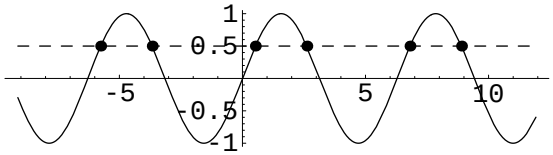
   $$\begin{aligned} (2x+6)(x+3) + 1 &= (3x-5)(x+3) \\ 2x^2 + 12x + 19 &= 3x^2 + 4x - 15 \\ 0 &= x^2 - 8x - 34 \\ x_1 = 4 + 5\sqrt{2} \quad &, \ x_2 = 4 - 5\sqrt{2} \end{aligned}$$

   (e) $\sqrt{2x-1} = 1 + \sqrt{x-1}$

   $$\begin{aligned} \left(\sqrt{2x-1}\right)^2 &= \left(1 + \sqrt{x-1}\right)^2 \\ 2x - 1 &= 1 + 2\sqrt{x-1} + (x-1) \\ x - 1 &= 2\sqrt{x-1} \\ x^2 - 2x + 1 &= 4(x-1) \\ x^2 - 6x + 5 &= 0 \\ x_1 = 1 \quad &, \ x_2 = 5 \end{aligned}$$

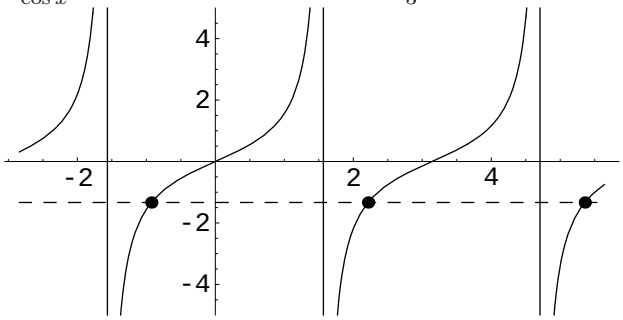
2. Lösen Sie folgende trigonometrische Gleichungen!

(a) $\sin x = 0.5 \quad \rightarrow \quad x_{k1} = \frac{\pi}{6} + 2k\pi, \quad x_{k2} = \pi - \frac{\pi}{6} + 2k\pi = \frac{5\pi}{6} + 2k\pi$



(b) $3\sin x + 4\cos x = 0 \mid : \cos x$

$3\frac{\sin x}{\cos x} + 4 = 0 \quad \rightarrow \quad \tan x = -\frac{4}{3} \quad \rightarrow \quad x_k = -0.927 + k\pi = -53.13° + k180°$

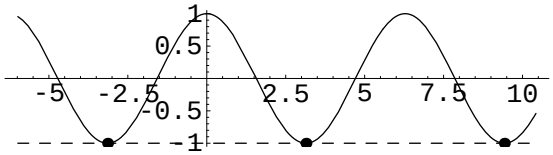


(c) $\cos^2 x - 2\sin^2 x - 6\cos x = \cos^2 x - 2(1 - \cos^2 x) - 6\cos x = 3\cos^2 x - 6\cos x - 2 = 7$

Setze $X = \cos x \quad \rightarrow \quad 3X^2 - 6X - 9 = 0 \quad \rightarrow \quad X^2 - 2X - 3 = 0$

$X_1 = 3$ entfällt, da $|\cos x| \leq 1$

$X_2 = -1 = \cos x \quad \rightarrow \quad x_{k1} = \pi + 2k\pi, \quad x_{k2} = -\pi + 2k\pi \quad \rightarrow \quad x_k = (2k+1)\pi$



3. Wenden Sie Potenz- und Wurzelgesetze an!

(a) $\frac{x^3\sqrt{x}}{x^5} = x^{3+\frac{1}{2}-5} = x^{-\frac{3}{2}} = \frac{1}{\sqrt{x^3}}$

(b) $\frac{a^{\frac{1}{2}}a^4}{\sqrt{a^5}} = a^{\frac{1}{2}+4-\frac{5}{2}} = a^2$

(c) $\sqrt[3]{a\sqrt{a^3}} = \left(a^1 a^{\frac{3}{2}}\right)^{\frac{1}{3}} = \left(a^{1+\frac{3}{2}}\right)^{\frac{1}{3}} = \left(a^{\frac{5}{2}}\right)^{\frac{1}{3}} = a^{\frac{5}{6}} = \sqrt[6]{a^5}$

4. Wenden Sie Logarithmengesetze an!

$\boxed{\log_a b = c \quad \rightarrow \quad a^c = b}$

(a) $\lg 0.001 = \lg 10^{-3} = -3\lg 10 = -3$

(b) $\lg x = 2 \quad \rightarrow \quad x = 10^2 = 100$

(c) $\ln x = 1 \quad \rightarrow \quad x = \mathrm{e}$

(d) $\log_2 2 = x \quad \rightarrow \quad 2^x = 2 \quad \rightarrow \quad x = 1$

(e) $\ln xy^3 = \ln x + \ln y^3 = \ln x + 3\ln y$

(f) $2\ln a - 3\ln b = \ln a^2 - \ln b^3 = \ln \frac{a^2}{b^3}$

(g) $\log_3 x = \frac{\ln x}{\ln 3} = \frac{\lg x}{\lg 3}$

(h) $\ln x\sqrt{x+3y} = \ln x + \frac{1}{2}\ln(x+3y)$

(e) $\ln y = 3 - \ln x | \mathrm{e}^{\cdots} \quad \rightarrow \quad y = \mathrm{e}^{3-\ln x} = \mathrm{e}^{3+\ln x^{-1}} = \mathrm{e}^3 \mathrm{e}^{\ln \frac{1}{x}} = \mathrm{e}^3 \frac{1}{x}$

(f) $\lg y = 1 + 2\lg x | 10^{\cdots} \quad \rightarrow \quad y = 10^{1+\lg x^2} = 10x^2$